

Probing an extended region of Δm^2 with rapidly oscillating ${}^7\text{Be}$ solar neutrinos

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(Received 29 June 2001; published 28 January 2002)

If ${}^7\text{Be}$ solar neutrinos can be observed in real time experiments, then an extended region of Δm^2 can be probed by a proper analysis of the rapidly changing phase of vacuum oscillations due to the eccentricity of the Earth's orbit about the Sun. For the case of maximal vacuum mixing, a kind of Fourier analysis of expected data for one year's time could uniquely pick out Δm^2 if it lies in the region $\sim 10^{-10} - (6 \times 10^{-9}) (\text{eV})^2$.

DOI: 10.1103/PhysRevD.65.053005

PACS number(s): 14.60.Pq, 13.10.+q, 25.30.Pt

I. INTRODUCTION

In a previous work it was argued [1] that with maximal vacuum mixing there is agreement, with minor modifications, between extant observations of solar neutrinos and predictions by the standard solar model (SSM) [2–5]. The maximal vacuum mixing case considered was that in which the phase of neutrino oscillations coming from the Sun is averaged, leading to 50% of the neutrinos arriving at Earth as electron neutrinos. As a result of this averaging, while $\sin^2 2\theta$ was assumed to be maximal (equal to one), Δm^2 was not determined and taken to lie in the approximate range $10^{-9} < \Delta m^2 \leq 10^{-3}$ with an exclusion of the approximate range $3 \times 10^{-7} < \Delta m^2 < 10^{-5}$ for maximal mixing [6] due to the lack of an observed day-night effect in the Super-Kamiokande data [7].

On the other hand, the recent first results of the SNO Collaboration measurement of charged current interactions produced by ${}^8\text{B}$ neutrinos [8], taken in combination with the elastic scattering result of the Super-Kamiokande Collaboration [7], indicate that only about one-third of the neutrinos arriving at Earth from the Sun are electron neutrinos, with the other two-thirds being μ or τ neutrinos. Oscillation into sterile neutrinos now seems relatively unlikely from the SNO result.

While at first glance this comparison seems to make maximal vacuum mixing less likely, a global analysis of the SNO result with the other solar neutrino experiments, chlorine [9], Super-Kamiokande [7], and gallium [10,11] has led to the conclusion that “the CC measurement by SNO has not changed qualitatively the globally allowed solution space for solar neutrinos, although the CC measurement has provided dramatic and convincing evidence for neutrino oscillations and has strengthened the case for active oscillations with large mixing angles [12].” Furthermore, global analyses [12,13] do not completely exclude solutions to the solar neutrino problem in the mass region $10^{-10} < \Delta m^2 < 10^{-8}$ for maximal (or near maximal) mixing. In the following, the time varying phase of oscillating ${}^7\text{Be}$ neutrinos is investigated as a possible method to discover (or exclude) a solution of the solar neutrino problem in that mass region.

In the mass region $4 \times 10^{-11} < \Delta m^2 < 10^{-9}$ there are so-called “just-so” vacuum solutions of the solar neutrino problem, where the phase of the oscillation of ${}^8\text{B}$ neutrinos coming from the Sun is not completely averaged [14,15]. Recall also [16,17], that there is a large change in the ${}^7\text{Be}$ electron

neutrino flux over the year in the ${}^8\text{B}$ “just-so” region due to the change in phase of order $\pi/2$ in a year brought about by the $\pm 1.67\%$ yearly orbital variation from the mean distance of the Sun to the Earth. As will be shown in the following when phase averaging due to the temperature of the Sun and phase damping due to the Mikheyev-Smirnov-Wolfenstein (MSW) effect are considered, it turns out that phase variation in ${}^7\text{Be}$ neutrinos should be observable for Δm^2 in the range from about 10^{-10} up to about $6 \times 10^{-9} (\text{eV})^2$. This observable range of Δm^2 via ${}^7\text{Be}$ neutrinos turns out to be in approximate agreement with a previous analysis by de Gouvêa, Friedland, and Murayama [18].

II. OSCILLATIONS: THERMAL AVERAGING; MSW DAMPING

There is a low-energy region of solar neutrinos dominated by the nearly monenergetic 862 keV line from electron capture by ${}^7\text{Be}$ in the Sun. It is the purpose of the Borexino [19] experiment, soon to come on line, to measure these neutrinos in real time. Feasibility studies are also being carried out for other experiments to measure ${}^7\text{Be}$ neutrinos in real time such as LENS [20] and HELLAZ [21].

If the ${}^7\text{Be}$ neutrinos were truly monoenergetic then the number of neutrinos detected via electron scattering in an experiment like Borexino (normalized to unity for no oscillations) would take the following form for vacuum oscillations:

$$R(\phi, \Delta m^2) = 1 - 0.79 \sin^2 2\theta \sin^2 \frac{\pi \Delta m^2 L(\phi)}{(0.862)(0.00248)}, \quad (1)$$

where θ is the vacuum mixing angle, Δm^2 is expressed in $(\text{eV})^2$, the μ or τ neutrino scattering relative to electron neutrino scattering at 0.862 MeV is 0.21 [22], and

$$L(\phi) = \frac{1 - \epsilon^2}{1 + \epsilon \cos \phi} 1.496 \times 10^8, \quad (2)$$

the distance from the Earth to the center of the Sun (in kilometers), which varies throughout the year due to the eccentricity $\epsilon = 0.0167$ of the Earth's orbit about the Sun. Note that since we take the number of neutrinos detected as a function of ϕ , the phase of the Earth in its orbit about the Sun, rather than of the time of the year, there is no $1/L^2$ seasonal varia-

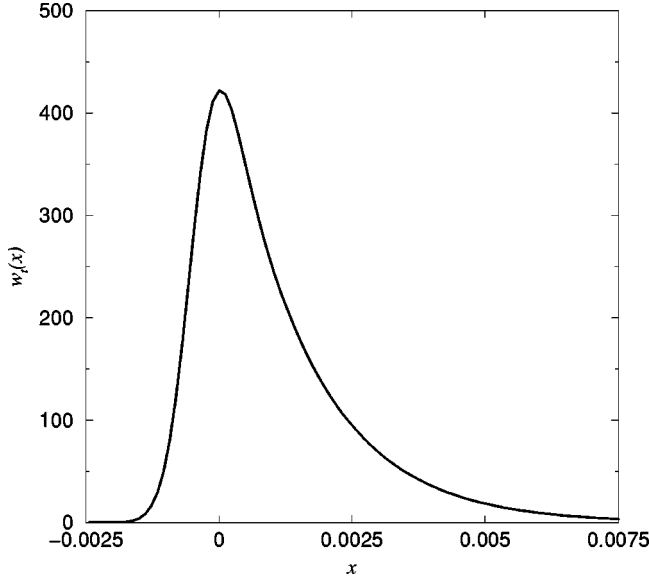


FIG. 1. Temperature broadening of ${}^7\text{Be}$ neutrinos originating in the Sun (see text).

tion in R ; it is canceled by the Jacobian in going from time as an independent variable to ϕ as an independent variable (Kepler's second law).

It has been pointed out by Pakvasa and Pantaleone [23] that the ${}^7\text{Be}$ energy line is thermally broadened not only by the spread in nuclear velocities (Doppler broadening) but also by the solar temperature of the approximately 80% of the capture electrons that come from the continuum. We make use of the published table of Bahcall [24] which was obtained by convoluting both these sources of thermal spreading to obtain the energy profile of the 862 keV ${}^7\text{Be}$ solar neutrino shown in Fig. 1. Note that the distribution $w_t(x)$ is asymmetric in shape and plotted as a function of $x = (E - 0.862)/0.862$ in MeV.

Expressing Δm^2 in $10^{-8} (\text{eV})^2$ one obtains

$$R(\phi, \Delta m^2) = 1 - 0.79 \sin^2 2\theta \int dx w_t(x) \sin^2 \left(\frac{2198 \Delta m^2}{(1 + 0.0167 \cos \phi)(1 + x)} \right), \quad (3)$$

or

$$R(\phi, \Delta m^2) = 1 - 0.395 \sin^2 2\theta \left[1 - \int dx w_t(x) \times \cos \left(\frac{4396 \Delta m^2}{(1 + 0.0167 \cos \phi)(1 + x)} \right) \right]. \quad (4)$$

For clarity and convenience we will retain these constants explicitly and express Δm^2 in units of $10^{-8} (\text{eV})^2$ unless otherwise specified for the rest of this paper.

Since x is constrained to contribute to Eq. (4) only when it is much smaller than unity, one may set $1/(1+x)$ equal to $1-x$ and carry out the integration to obtain

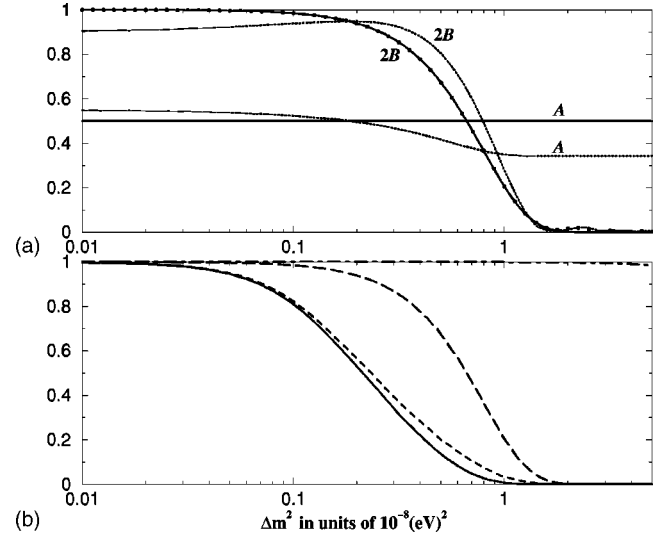


FIG. 2. Top panel: Constant A and oscillating B part of electron neutrino rate emerging from the Sun for $\sin^2 2\theta = 1$ (solid line) and for $\sin^2 2\theta = 0.9$ (dotted line). Bottom panel: Total oscillation damping factor (solid line) and partial damping factors (see text).

$$R(\phi, \Delta m^2) = 1 - 0.395 \sin^2 2\theta \left[1 - I_c \left(\frac{\Delta m^2}{1 + 0.0167 \cos \phi} \right) \times \cos \left(\frac{4396 \Delta m^2}{1 + 0.0167 \cos \phi} \right) - I_s \left(\frac{\Delta m^2}{1 + 0.0167 \cos \phi} \right) \times \sin \left(\frac{4396 \Delta m^2}{1 + 0.0167 \cos \phi} \right) \right], \quad (5)$$

with

$$I_c(u) = \int dx w_t(x) \cos(ux), \quad I_s(u) = \int dx w_t(x) \sin(ux). \quad (6)$$

In addition to temperature damping of the oscillations there is a second factor that we may call “(MSW) damping.” With maximal mixing, the phase-averaged rate of electron neutrinos does not depend on whether a MSW transition has taken place. However, if an MSW transition has taken place in the Sun, then the maximally mixed neutrino emerges in the form of a pure mass eigenstate [i.e., $A_1 = 0$, $A_2 = 1$ in Eq. (A5) of the Appendix]. Although the pure mass eigenstate $|\nu_2\rangle$ is half electron neutrino and half other flavor neutrino, there is no interference from Eq. (A5) and thus no oscillation. The probability of remaining an electron neutrino remains one-half without variation in the vacuum from the Sun to the Earth. In contrast, pure vacuum oscillations with maximal mixing (no MSW transition) leads to equal parts of each mass eigenstate; the neutrinos oscillate from pure electron neutrino to pure other flavor neutrino on the path from the Sun to the Earth. However, the phase-averaged probability of an electron neutrino reaching the Earth is still one-half. Guth,

Randall, and Serna [25] have pointed out the relevance of this difference for matter oscillations in the Earth: there can be a day-night effect, even for the case of maximal mixing if there has been an MSW transition in the Sun. The Appendix comprises a short digression on this point. The treatment in the Appendix assumes phase averaging over the distances involved, due to the larger Δm^2 values that would come into play in a possible day-night effect. Here we are interested in the phase of the vacuum oscillation, since that is our signal.

The onset of MSW conversion in the Sun with larger Δm^2 can be investigated numerically by utilizing a piece of computer code adapted from a previous investigation [26]. The rate of ${}^7\text{Be}$ electron neutrinos emerging from the Sun (again normalized to unity for no oscillations) takes the form

$$R(\Delta m^2) = A + B \cos \frac{2\pi \Delta m^2 X}{(0.862)0.00248}, \quad (7)$$

with X the distance from the surface of the Sun plus some constant. For maximal mixing, $A=0.5$ and B is 0.5 for vacuum oscillations but B vanishes for complete adiabatic conversion.

The top panel in Fig. 2 shows how the magnitude of the oscillation for maximum mixing is reduced with increasing Δm^2 even though the phase-averaged mixing remains a constant. The filled circles are the values of $2B$ calculated numerically for $\sin^2 2\theta=1$. The solid line through these circles approximates $2B$ by $\exp[-1.583(\Delta m^2)^2]$. Except for the small oscillations beyond $\Delta m^2=10^{-8}$, this ‘‘MSW damping’’ is well represented by the Gaussian factor. The solid line at 0.5 represents A for maximal mixing. The dotted lines are the corresponding quantities $2B$ and A for $\sin^2 2\theta=0.9$, just for comparison.

Incorporating MSW damping Eq. (5) then becomes

$$R(\phi, \Delta m^2) = 1 - 0.395 \sin^2 2\theta \left[1 - \exp[-1.583(\Delta m^2)^2] \left\{ I_c \left(\frac{\Delta m^2}{1 + 0.0167 \cos \phi} \right) \cos \left(\frac{4396 \Delta m^2}{1 + 0.0167 \cos \phi} \right) + I_s \left(\frac{\Delta m^2}{1 + 0.0167 \cos \phi} \right) \sin \left(\frac{4396 \Delta m^2}{1 + 0.0167 \cos \phi} \right) \right\} \right]. \quad (8)$$

This is the expression that we use in the calculations to follow.

Equation (8) may also be written in the form

$$R(\phi, \Delta m^2) = 1 - 0.395 \sin^2 2\theta \left[1 - \exp[-1.583(\Delta m^2)^2] I_b \left(\frac{\Delta m^2}{1 + 0.0167 \cos \phi} \right) \cos \left(\frac{4396 \Delta m^2}{1 + 0.0167 \cos \phi} - \delta \right) \right], \quad (9)$$

where

$$\delta = \arctan \frac{I_s}{I_c}, \quad (10)$$

and

$$I_b = I_c \sec \delta. \quad (11)$$

For the purpose of illustration we ignore the $\cos \phi$ dependence in the temperature damping factor and consider $I_b(\Delta m^2)$. The bottom panel of Fig. 2 shows $I_b(\Delta m^2)$ as the short-dashed line and repeats the Gaussian MSW damping factor from the above panel as the long-dashed line. The solid line is the product of the two, the overall damping factor including temperature spreading and MSW damping. It is clear that there is a complete damping out of the oscillations at $\Delta m^2 \sim 10^{-8} (\text{eV})^2$, and that this broadening averages out the phase of the oscillations at higher values of Δm^2 .

Finally one should note that the source broadening is insignificant because of the following. It turns out that the SSM density [4] of ${}^7\text{Be}$ neutrinos produced as a function of the solar radius is very close to a Gaussian function of the Sun’s radius,

$$w_s(r) = \sqrt{\frac{\beta}{\pi}} e^{-\beta r^2}, \quad (12)$$

where $\beta = 13.166 \times 10^6$, and r is the distance from the center of the Sun in units of the distance from the Earth to the Sun. However, as has been pointed out [27,18], the oscillations effectively start not at the source but at the level crossing point. For the present maximal mixing case the level crossing point is at surface of the Sun. The neutrinos originating off the Sun’s axis in the direction of the Earth will have a slightly larger distance to travel due to the curvature of the Sun’s surface. The Gaussian density Eq. (12) leads, in a very good approximation, to a source spreading density of the form

$$w_s(z) = \mu e^{-\mu z}, \quad (13)$$

where $\mu = 1.23 \times 10^5$ or twice β times the ratio of the Sun’s radius to the mean Earth-Sun distance, and z is the distance from the point on the Sun’s surface closest to Earth toward its center. If this small source broadening were the only cause of damping, then by an analytical treatment paralleling that leading to Eq. (9), one would find a source damping factor

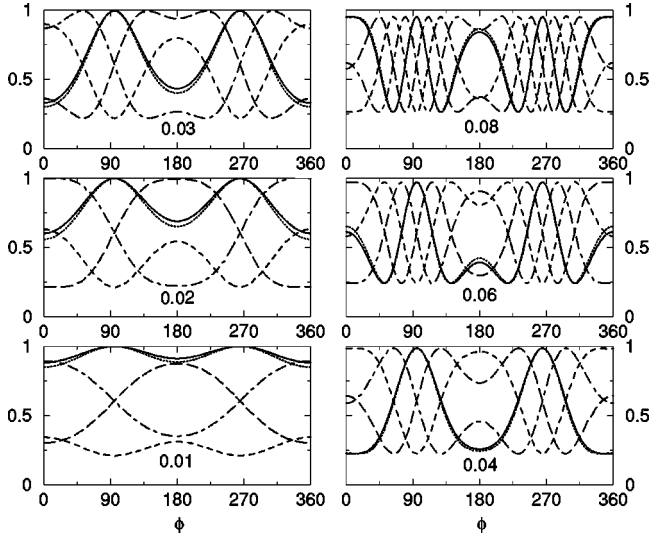


FIG. 3. Rate of ${}^7\text{Be}$ neutrinos detected by electron scattering for maximal mixing. The number on each panel gives Δm^2 for the solid line. The long and short dashed line is Δm^2 plus approximately 0.00035, the short dashed line is Δm^2 plus 0.0007, the long dashed line is Δm^2 plus 0.00105, and the dotted line is Δm^2 plus 0.0014.

$$I(\Delta m^2) = \frac{1}{\sqrt{1 + (4396\Delta m^2/\mu)^2}} = \frac{1}{\sqrt{1 + 0.001277(\Delta m^2)^2}}. \quad (14)$$

The long and short dashed line in the bottom panel of Fig. 2 represents $I(\Delta m^2)$. Obviously $I(\Delta m^2)$ only starts to deviate from unity at the rightmost part of the plot (at $\Delta m^2 = 5$). Thus, source broadening is insignificant for our region of interest.

The region that we will investigate spans the range from $\Delta m^2 \sim 10^{-10}$, the “just-so” region for ${}^8\text{B}$, up to $\Delta m^2 \sim 10^{-8}$, where the broadening averages the phase. As noted above and in the Appendix, one might in principle begin to see a day-night effect [28] with the onset of MSW damping. In fact there would be a sizable day-night effect for maximal mixing at $\Delta m^2 \sim 10^{-7}$ [29,26] (the so called “low” MSW solution).

Figure 3 shows $R(\phi, \Delta m^2)$ for maximal mixing $\sin^2 2\theta = 1$ beginning at the low end with $\Delta m^2 = 0.01$ [again in units of $10^{-8} (\text{eV})^2$]. Note from Eq. (8) that the overall phase of the cosine factor depends on Δm^2 and that this phase changes by 180° when Δm^2 changes by $\pi/4396 \approx 0.000714$. This phase sensitivity is illustrated Fig. 3 where for each panel in addition to the curve for the labeled value of Δm^2 there are also curves for that value plus the appropriate increments to shift the overall phase by 90° , 180° , 270° , and 360° . Figure 4 shows the increasing frequency of the oscillations of $R(\phi, \Delta m^2)$ as a function of ϕ for the Δm^2 region of 10^{-9} to 10^{-8} . Note also the decreasing amplitude of the oscillations as they come close to being damped out by the temperature plus source broadening and MSW damping at 0.8×10^{-8} .

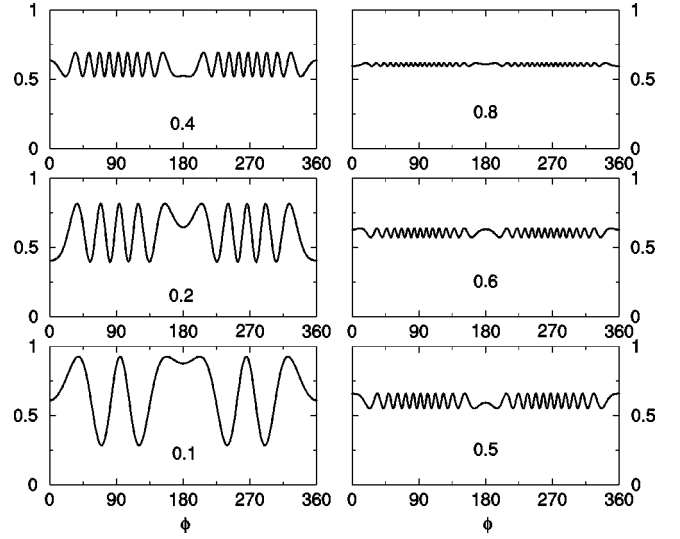


FIG. 4. Rate of ${}^7\text{Be}$ neutrinos detected by electron scattering for maximal mixing. The number on each panel gives Δm^2 for the solid line.

III. OSCILLATIONS: ANALYSIS OF THE SIGNAL

With such a rapid oscillation period throughout the year seen especially in Fig. 4 (on the order of several months to several days), one anticipates that for such values of Δm^2 there would be insufficient statistics at an experiment like Borexino for a pattern to be obvious. However, in what follows we will investigate how a Fourier-type analysis of data from such experiments could give evidence of a phased oscillation and thereby determine the value of Δm^2 if it lies in this range. Fourier analyses of ${}^7\text{Be}$ solar neutrino data have been previously proposed [30,31], but what follows here is a somewhat different approach.

Since Borexino is a detector rather than radiochemical experiment, it records the information on when each count was recorded and thereby the distance of the detector to the Sun L incorporated as ϕ in Eq. (8). We suggest analyzing data from such experiments by effectively integrating data with a factor $\exp \exp[i4396\Delta m^2/(1 + 0.0167 \cos \phi)]$ and varying Δm^2 over the range $\sim 10^{-10}$ – 10^{-8} to look for a signal.

To test whether the Δm^2 can be determined by such a method, Monte Carlo data sets have been simulated in the following way. Random numbers are generated uniformly for ϕ from 0 to 2π in order to cover the year and make use of Eq. (8). In order to weight events according to what would be expected from Eq. (8) with a specific Δm^2 , a second random number between 0 and 1 is then generated for each ϕ and a count is generated if the random number is less than $R(\phi, \Delta m^2)$. This is the data set: the collection of specific angles, $\{\phi_i\}$, at which single events are recorded during a year.

Figure 5 shows a sample analysis of a data set generated from 15000 Monte Carlo attempts for $\Delta m^2 = 0.3$ in our units. The top panel shows the expected oscillation pattern, $R(\phi, 0.3)$. From Eq. (8) one would expect about 9075 data points to lie below the curve from 15000 random attempts,

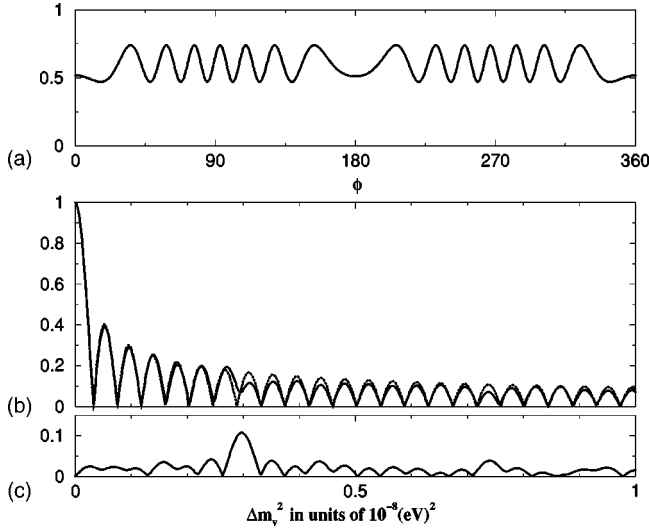


FIG. 5. The top panel gives the rate of ${}^7\text{Be}$ neutrinos coming from the Sun for maximal mixing and $\Delta m^2=0.3$. The solid line in the middle panel gives the absolute value of the Fourier analysis of the distribution $|I(0.5, \Delta m_v^2)|$ and the dashed line gives the absolute value of the analysis of the constant average of the distribution $|J(\Delta m_v^2)|$. The bottom panel shows the absolute value of the difference of the Fourier analyses of the distribution and the constant average of the distribution $|K(0.5, \Delta m_v^2)|$. This is the quantity that extracts the Δm^2 signal.

and in fact a set of 8993 data points $\{\phi_i\}$ were generated in this sample. The number of data points in this sample corresponds roughly to a year's running time at Borexino. For analysis one might first consider a Fourier type transformation on the set $\{\phi_i\}$:

$$\begin{aligned} I(\Delta m^2, \Delta m_v^2) &= \frac{1}{n} \sum_{i=1,n} \phi_i \exp\left(\frac{i4396\Delta m_v^2}{1+0.0167 \cos \phi_i}\right) \\ &\equiv \frac{1}{2\pi} \int_0^{2\pi} R(\phi, \Delta m^2) \exp\left(\frac{i4396\Delta m_v^2}{1+0.0167 \cos \phi}\right) \\ &\quad \times d\phi. \end{aligned} \quad (15)$$

The solid curve in the middle panel displays $|I(0.3, \Delta m_v^2)|$ and it shows a discontinuity in pattern near $\Delta m_v^2=0.3$. If there were no phase oscillation then one would expect I to approach a function that we will call $J(\Delta m_v^2)$:

$$J(\Delta m_v^2) = \frac{1}{2\pi} \int_0^{2\pi} \exp\left(\frac{i4396\Delta m_v^2}{1+0.0167 \cos \phi}\right) d\phi. \quad (16)$$

The dotted line in the middle panel displays $|J(\Delta m_v^2)|$. This suggests that we subtract off the Bessel-function-like behavior $J(\Delta m_v^2)$ contained in $I(\Delta m^2, \Delta m_v^2)$ (which tends to obscure the signal), create a new function

$$K(\Delta m^2, \Delta m_v^2) = I(\Delta m^2, \Delta m_v^2) - J(\Delta m_v^2), \quad (17)$$

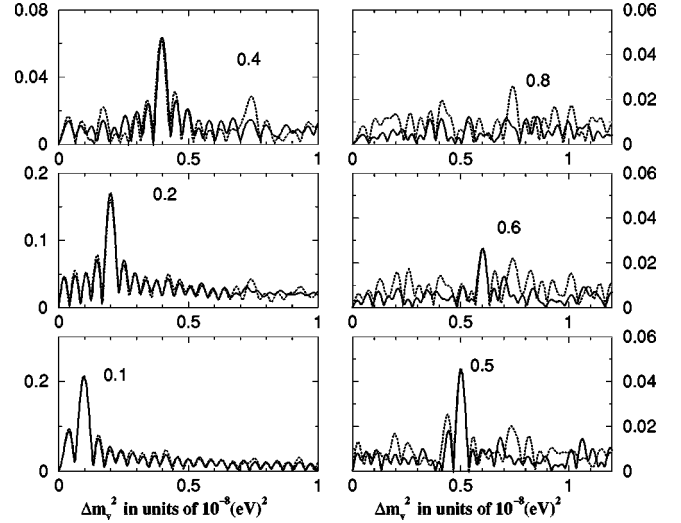


FIG. 6. Extraction of the signal $|K(\Delta m^2, \Delta m_v^2)|$. The number on each panel gives the value of Δm^2 . The dotted lines represent 15 000 tries or about 9000 events. The solid lines represent four times the statistics: 60 000 tries or about 36 000 events.

and use $|K(\Delta m^2, \Delta m_v^2)|$ to analyze our data set $\{\phi_i\}$. $|K(0.3, \Delta m_v^2)|$ is displayed in the bottom panel. The signal of $\Delta m^2=0.3$ is unambiguous.

Figure 6 shows that an unambiguous signal would be obtained with about one year's Borexino statistics for Δm^2 in the range 0.1–0.5. For $\Delta m^2=0.6$ there is a signal at $\Delta m_v^2=0.6$ that might be a little ambiguous with only one year's statistics, but it retains its shape with increasing statistics; a secondary peak seen at about $\Delta m_v^2=0.75$ with one year's statistics goes away with the higher statistics. There is no signal apparent for the $\Delta m^2=0.8$ case, as one would expect from looking at the corresponding curve in Fig. 4.

Figure 7 shows the extraction of the signal for Δm^2 an order of magnitude lower. Curves correspond to values in Fig. 3. Below $\Delta m^2=0.04$ this particular analysis starts to become ambiguous. However, the slow rate of variation with ϕ makes direct comparison with the patterns seen in Fig. 3 practical. At $\Delta m^2=0.01$ the analysis is complicated by the large change in magnitude of the rate throughout the year with a small increment in the value of Δm^2 .

IV. DISCUSSION

Based on the SNO result it now seems likely that the solar neutrino puzzle has been solved. It is not a deficiency in the standard solar model that is being observed but new physics. Electron neutrinos are oscillating into some combination of μ and τ neutrinos. Exactly how this happens is perhaps not yet clear, whether by one of the MSW solutions or some vacuum mixing solution. In the preceding sections of this paper it has been shown that if the solution to the solar neutrino puzzle happens to be maximal mixing in the mass range $\Delta m^2 = \sim 10^{-10} - (6 \times 10^{-9}) \text{ eV}^2$, then a proper analysis of a successful ${}^7\text{Be}$ neutrino experiment should be able to unambiguously determine Δm^2 . Not seeing a Δm^2 signal in this

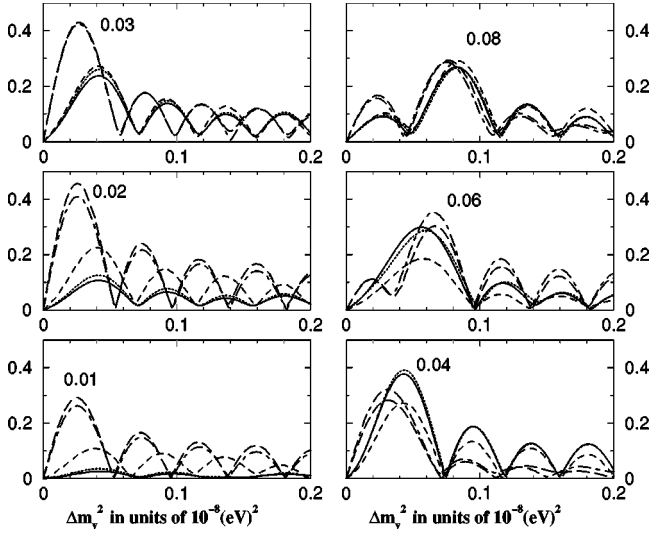


FIG. 7. $|K(\Delta m^2, \Delta m^2)|$ as in Fig. 6, but for lower values of Δm^2 . The location of the peak becomes less well defined with decreasing Δm^2 . As in Fig. 3 the long and short dashed line is Δm^2 plus approximately 0.00035, the short dashed line is Δm^2 plus 0.0007, the long dashed line is Δm^2 plus 0.00105, and the dotted line is Δm^2 plus 0.0014.

mass range would eliminate a region of Δm^2 for large mixing angle.

ACKNOWLEDGMENTS

I would like to thank Chellis Chasman for reading and commenting on the manuscript. This manuscript has been authored under Contract No. DE-AC02-98CH10886 with the U. S. Department of Energy.

APPENDIX: THE DAY-NIGHT EFFECT IN THE LIMIT OF MAXIMAL TWO NEUTRINO MIXING

Guth, Randall, and Serna [25] have pointed out that there can be a day-night effect, even for the case of maximal mixing. What follows is a compact explication of this point with emphasis on the limits of no MSW and maximal MSW effect in the Sun.

The general form for two mass eigenstates in two neutrino mixing is

$$|\nu_1\rangle = \cos\theta |\nu_e\rangle + \sin\theta |\nu_x\rangle, \quad (\text{A1})$$

and

$$|\nu_2\rangle = -\sin\theta |\nu_e\rangle + \cos\theta |\nu_x\rangle, \quad (\text{A2})$$

where $|\nu_x\rangle$ is presumed to be some linear combination of $|\nu_\mu\rangle$ and $|\nu_\tau\rangle$. Conversely

$$|\nu_e\rangle = \cos\theta |\nu_1\rangle - \sin\theta |\nu_2\rangle, \quad (\text{A3})$$

and

$$|\nu_x\rangle = \sin\theta |\nu_1\rangle + \cos\theta |\nu_2\rangle. \quad (\text{A4})$$

In free-space mass eigenstates propagate independently. A mixed mass state $|\nu(t)\rangle$ then has the form

$$|\nu(t)\rangle = e^{-im_1^2 t/2E} A_1 |\nu_1\rangle + e^{-im_2^2 t/2E} A_2 |\nu_2\rangle. \quad (\text{A5})$$

The probability P_s that a neutrino born in the Sun is an electron neutrino when it reaches Earth is then

$$P_s = \cos^2\theta P_1 + \sin^2\theta P_2 \quad (\text{A6})$$

with $P_{1,2} = |A_{1,2}|^2$ the average probability of a mass one or mass two eigenstate arriving at Earth where the phase has been averaged by the distances involved. Since $P_1 = 1 - P_2$ this may also be written equivalently

$$P_2 = \frac{(\cos^2\theta - P_s)}{\cos 2\theta}. \quad (\text{A7})$$

The probability P that an electron neutrino born in the Sun will be an electron neutrino after passing through the Sun, traveling to Earth, and then passing through Earth is simply

$$P = P_2 P_{2e} + (1 - P_2)(1 - P_{2e}), \quad (\text{A8})$$

where P_{2e} is the probability that a mass 2 neutrino entering Earth emerges at the detector as an electron neutrino. Making use of Eq. (A7) this becomes

$$P = P_s + \frac{(1 - 2P_s)}{\cos 2\theta} (P_{2e} - \sin^2\theta). \quad (\text{A9})$$

This expression is the Mikeyev-Smirnov expression [32] for the day-night effect, trivially transformed [26] to be most transparent in various limits.

The maximum value for P_s occurs for vacuum oscillations

$$P_s = 1 - \frac{1}{2} \sin^2 2\theta \quad (\text{A10})$$

and

$$\frac{(1 - 2P_s)}{\cos 2\theta} = -\cos 2\theta. \quad (\text{A11})$$

The minimum value for P_s occurs for complete adiabatic MSW conversion to a pure mass eigenstate $|\nu_2\rangle$. In this case from Eq. (A6)

$$P_s = \sin^2\theta \quad (\text{A12})$$

and

$$\frac{(1 - 2P_s)}{\cos 2\theta} = 1. \quad (\text{A13})$$

Thus as $\sin 2\theta$ goes to 1 (maximal mixing) there is no day-night effect for vacuum oscillations and a maximum effect possible in the case of complete adiabatic conversion.

- [1] Anthony J. Baltz, Alfred Scharff Goldhaber, and Maurice Goldhaber, Phys. Rev. Lett. **81**, 5730 (1998).
- [2] J. N. Bahcall and R. N. Ulrich, Rev. Mod. Phys. **60**, 297 (1988).
- [3] J. N. Bahcall and M. H. Pinsonneault, Rev. Mod. Phys. **64**, 885 (1992).
- [4] John N. Bahcall, M. Pinsonneault, and G. J. Wasserburg, Rev. Mod. Phys. **67**, 781 (1995).
- [5] J. N. Bahcall, S. Basu, and M. H. Pinsonneault, Phys. Lett. B **433**, 1 (1998).
- [6] J. N. Bahcall, P. I. Krastev, and A. Yu. Smirnov, Phys. Rev. D **58**, 096016 (1998).
- [7] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1158 (1998).
- [8] SNO Collaboration, Q. R. Ahmed *et al.*, Phys. Rev. Lett. **87**, 071301 (2001).
- [9] B. T. Cleveland *et al.*, Astrophys. J. **496**, 505 (1998).
- [10] GALLEX Collaboration, W. Hampel *et al.*, Phys. Lett. B **447**, 127 (1999).
- [11] SAGE Collaboration, J. N. Abdurashitov *et al.*, Phys. Rev. C **60**, 055801 (1999).
- [12] John N. Bahcall, M. C. Gonzalez-Garcia, and Carlos Peña-Garay, J. High Energy Phys. **08**, 014 (2001).
- [13] G. L. Fogli, E. Lisi, D. Mantanino, and A. Palazzo, Phys. Rev. D **64**, 093007 (2001).
- [14] N. Hata and P. G. Langacker, Phys. Rev. D **56**, 6107 (1997).
- [15] James M. Gelb and S. P. Rosen, Phys. Rev. D **60**, 011301 (1999).
- [16] P. I. Krastev and S. T. Petcov, Nucl. Phys. **B449**, 605 (1995).
- [17] James M. Gelb and S. P. Rosen, hep-ph/9908325.
- [18] André de Gouvêa, Alexander Friedland, and Hitoshi Murayama, Phys. Rev. D **60**, 093011 (1999).
- [19] Borexino Collaboration, G. Alimonti *et al.*, hep-ex/0012030.
- [20] LENS Collaboration, M. Cribier, Nucl. Phys. B (Proc. Suppl.) **87**, 195 (2000).
- [21] P. Gorodetzky, A. de Bellefon, J. Dolbeau, T. Patzak, P. Salin, A. Sarrat, and J. C. Vanel, Nucl. Phys. B (Proc. Suppl.) **87**, 506 (2000).
- [22] J. N. Bahcall, *Neutrino Astrophysics* (Cambridge University Press, Cambridge, England, 1989).
- [23] Sandip Pakvasa and James Pantaleone, Phys. Rev. Lett. **65**, 2479 (1990).
- [24] John N. Bahcall, Phys. Rev. D **49**, 3923 (1994).
- [25] Alan H. Guth, Lisa Randall, and Mario Serna, J. High Energy Phys. **08**, 018 (1999).
- [26] A. J. Baltz and J. Weneser, Phys. Rev. D **50**, 5971 (1994).
- [27] J. Pantaleone, Phys. Lett. B **251**, 618 (1990).
- [28] A. J. Baltz and J. Weneser, Phys. Rev. D **35**, 528 (1987); **37**, 3364 (1988); E. D. Carlson, *ibid.* **34**, 1454 (1986); J. Bouchez, M. Cribier, W. Hampel, J. Rich, M. Spiro, and D. Vignaud, Z. Phys. C **32**, 499 (1986).
- [29] R. S. Raghavan, A. B. Balantekin, F. Loreti, A. J. Baltz, S. Pakvasa, and J. Pantaleone, Phys. Rev. D **44**, 3786 (1991).
- [30] A. De Rujula and S. Glashow, CERN-TH 6655/92 (1992).
- [31] G. L. Fogli, E. Lisi, and D. Mantanino, Phys. Rev. D **56**, 4374 (1997).
- [32] S. P. Mikheyev and A. Yu. Smirnov, in *New and Exotic Phenomena*, Proceedings of the Moriond Workshop, Les Arcs, Savoie, France, 1987, edited by O. Fackler and J. Tran Thanh Van (Editions Frontieres, Gif-sur-Yvette, France, 1987), p. 405.